Strong and electromagnetic decays for excited heavy mesons

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Abstract

We discuss a model for heavy mesons where the light quark (u or d) moves in the colour electric field from a heavy quark (c or b) placed in the center of the bag. We calculate energy spectra for pionic and photonic transitions from excited states. The transition amplitudes and the branching ratios between electromagnetic and pionic transitions compares favorable with the limited amount of known experimental data.

1 Introduction

Heavy quark spectroscopy is a very interesting and rewarding subject for study. The discovery of charmonium definitely swept away all doubts the physics community had about the existence of quarks as the fundamental building blocks of hadrons.

Mesons with one heavy and one light quark is a further excellent laboratory to test our ideas about strong interactions. These mesons are in a way the hydrogen atoms of quark physics. As the mass of the heavy quark increases, its motion become gradually less and less important and the physical properties of the heavy-light, $Q\overline{q}$, meson are more and more determined by the dynamics of the light quark.

The discovery of the heavy quark symmetries by Isgur and Wise [1, 2] and the creation of a heavy quark effective theory from QCD[3, 4, 5] has been extremely important for the analysis of the physics of heavy hadrons [6]

Ideally one would like to compute the couplings in the baryon and meson Lagrangian from QCD - in time this should be provided by lattice QCD calculations. In the meantime model calculations can be useful and one might hope that these give us some physical insight for the long distance behavior of the quark interaction.

2 The model

There are many models used in quark physics and we have chosen a variant of the M.I.T. bag model that was created by W. Wilcox, O. V. Maxwell and K. A. Milton [7] (WWM), at a time when there were little information about exited systems made of one heavy and one light quark.

The model is a nice theoretical laboratory, it lends itself to analytical calculations and it seems to give results that are not too far from experimental results. In particular it seems to work well for calculations of the Isgur-Wise function [8] and to represents an improvement over results coming from the M.I.T. bag model [9]. In the WMM-model the heavy quark is placed in the center of the bag and the light quark moves in the colour electromagnetic field set up by the heavy quark.

The Hamiltonian for the light quark is then

$$H = H_0 + H_I \qquad \text{where} \tag{1}$$

$$H_0 = \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m + g\boldsymbol{t}_{la}V^a, \ \ \boldsymbol{t}_a \equiv \frac{\boldsymbol{\lambda_a}}{2} \ \ a = 1, \dots , 8 \quad \text{and} \tag{2}$$

$$H_I = -g\boldsymbol{t}_{la}\boldsymbol{\alpha} \cdot \boldsymbol{A}^a. \tag{3}$$

$\boldsymbol{t}_a$ are the generators of the $SU(3)_C$ colour group. The index $l(h)$ refers to the light(heavy) quark. We have used $A^\mu_a = (V_a, \boldsymbol{A}_a)$ and the usual notation $\alpha_i = \gamma^0\gamma_i$ and $\beta = \gamma^0$, where the γ's are the Dirac matrices and $\boldsymbol{\lambda}_a$ the Gellman matrices. V_a and $\boldsymbol{A_a}$ are the colour electric potentials and vector fields respectively produced by the heavy quark.

As gluon selfcouplings are neglected and the heavy quark is treated as point like the potential has a Coulomb like form :

$$V_a = \frac{g\boldsymbol{t}_{ha}}{4\pi r} \tag{4}$$

Substituting this potential into equation (2) give us :

$$H_0 = \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m + \frac{g^2\boldsymbol{t}_l^a\boldsymbol{t}_{ha}}{4\pi r}. \tag{5}$$

Using the constraint that the meson is a colour singlet, that is $\boldsymbol{t}_l^a\boldsymbol{t}_{ha} = -4/3$ the equation of motion of the light quark in the meson rest frame is then simply

$$H_0 = \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m - \frac{\xi}{r}, \tag{6}$$

where $\xi = \frac{4}{3}\alpha_s = \frac{4}{3}\frac{g^2}{4\pi}$.

The four component wave function $\psi(r)$ of the light quark (ignoring H_I) is therefore the well known solutions for the relativistic Coulomb problem. We shall use the notation

$$\psi(r) = \begin{pmatrix} g(r)\chi^\mu_\kappa \\ if(r)\chi^\mu_{-\kappa} \end{pmatrix}, \tag{7}$$

where χ^μ_κ is the two component spinors describing the angular part of the wavefunction.

The energy of the confined light quark is determined by the Bogolioubov-MIT boundary condition [10, 11] which in the rest frame of the meson takes the form $-i(\hat{\boldsymbol{r}} \cdot \boldsymbol{\gamma})\psi = \psi$. Substituting equation (7) into this equation and using the following property $(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}})\chi^\mu_\kappa = -\chi^\mu_{-\kappa}$ give us :

$$f(R) + g(R) = 0, \tag{8}$$

where R is the radius of the spherical bag. The confinement of the light quark presumably originating from the gluonic selfcouplings is now taken care of by equation (8) and the surface conditions :

$$\hat{\boldsymbol{r}} \cdot \boldsymbol{E}^a = 0 \tag{9}$$

$$\hat{\boldsymbol{r}} \times \boldsymbol{B}^a = 0 \tag{10}$$

The vector fields $\boldsymbol{A}_a$ that are set up by the heavy quark and fulfill the boundary conditions (10) are

$$\boldsymbol{A}_a = \frac{1}{4\pi}(\frac{\boldsymbol{m}_a \times \hat{\boldsymbol{r}}}{r^3} + \frac{\boldsymbol{m}_a \times \hat{\boldsymbol{r}}}{2R^3}), \tag{11}$$

where $\boldsymbol{m}_a$ is the colour magnetic moment(s) of the heavy quark :

$$\boldsymbol{m}_a = \frac{g\boldsymbol{t}_{ha}}{M_h}\boldsymbol{S}. \tag{12}$$

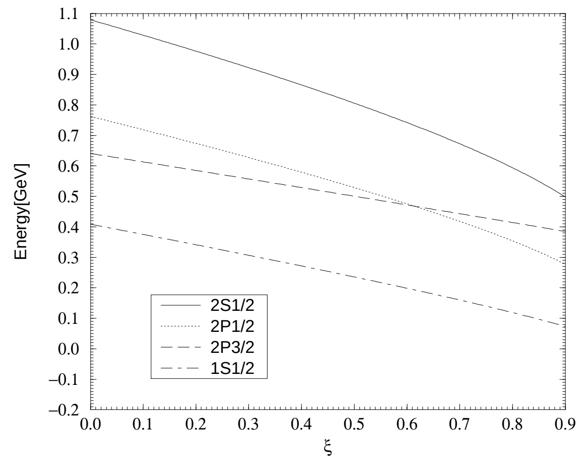

Figure 1: Energy levels inside a bag with radius $R = 5GeV^{-1}$.

M_h is the mass of the heavy quark and $\boldsymbol{S}$ its spin operator. H_I now takes the form :

$$H_I = \frac{\alpha_s \boldsymbol{t}_{la} \boldsymbol{t}_h{}^a}{M_h} \boldsymbol{S} \cdot (\boldsymbol{\alpha} \times \hat{\boldsymbol{r}})(\frac{1}{r^2} + \frac{r}{2R^3}) \tag{13}$$

The contribution of H_I to the energy is calculated perturbatively and the hyperfine splitting energy to first order in α_s is

$$E_I^1 = \frac{8}{3}\frac{\alpha_s}{M}\frac{\kappa}{4\kappa^2 - 1}(F(F+1) - J(J+1) - \frac{3}{4})\frac{1}{N}\int_0^R dr(2 + \frac{r^3}{R^3})f(r)^* g(r). \tag{14}$$

N is the normalization of the wavefunction, $N \equiv \int_0^R drr^2(|f(r)|^2 + |g(r)|^2)$. Here F is the total angular momentum of the mesonic system and J is the light quark (total) angular momentum. From equation (14) we see for $M_h \to \infty$ then $E_I^1 \to 0$; this is the heavy quark limit.

The mass functional for a heavy meson described in our bag model will be :

$$M = M(R) = E_{Vol} + E_{Zero} + m_Q + E_q \tag{15}$$

where $E_{Vol} = \frac{4\pi}{3}BR^3$ is the energy needed to create a bag in vacuum , E_{Zero} is the zero point energy proportional to $1/R$, m_Q is the heavy quark mass and E_q is the light quark energy $E_q \equiv \sqrt{p_q^2 + m_q^2} + E_I^1$, where $E_I^1$is the hyperfine splitting energy to first order given in equation (14). We will say more about the first two terms later. For now we only note that if the radii of two mesons with same flavour of the heavy quark is kept constant, then the mass difference between them are given by the following formula :

$$\Delta M = E_q(nL_J) - E_q(n'L'_{J'}). \tag{16}$$

This means that if the radii of two mesons do not differ too much, then the difference between the energy levels is directly related to the mass difference of the two mesons.

It is of interest first to see how the energy levels of the meson are ordered in the heavy quark limit when the colour electric central potential increases, these are shown in fig. 1.

As we can see an increase in the central field from the special case where the light quark moves freely in the bag, reduces the mass of the P states - and even make them cross. The odd parity states stay however roughly half way between the ground state and the first excited S-state. This will lead to some difficulties when we try to fit the spectrum of heavy meson states.

The breaking of the heavy quark limit is given by the spin-spin interaction and in our model by the term E_I^1, given in equation (14). It should be noted that this term is dependent of both $1/M$ and α_s which determines the strength of the central four vector potential the light quark moves in. In this respect our model is more constraining than most models where the interquark central potential is unrelated to the strength of the spin-spin and spin-orbit interaction.

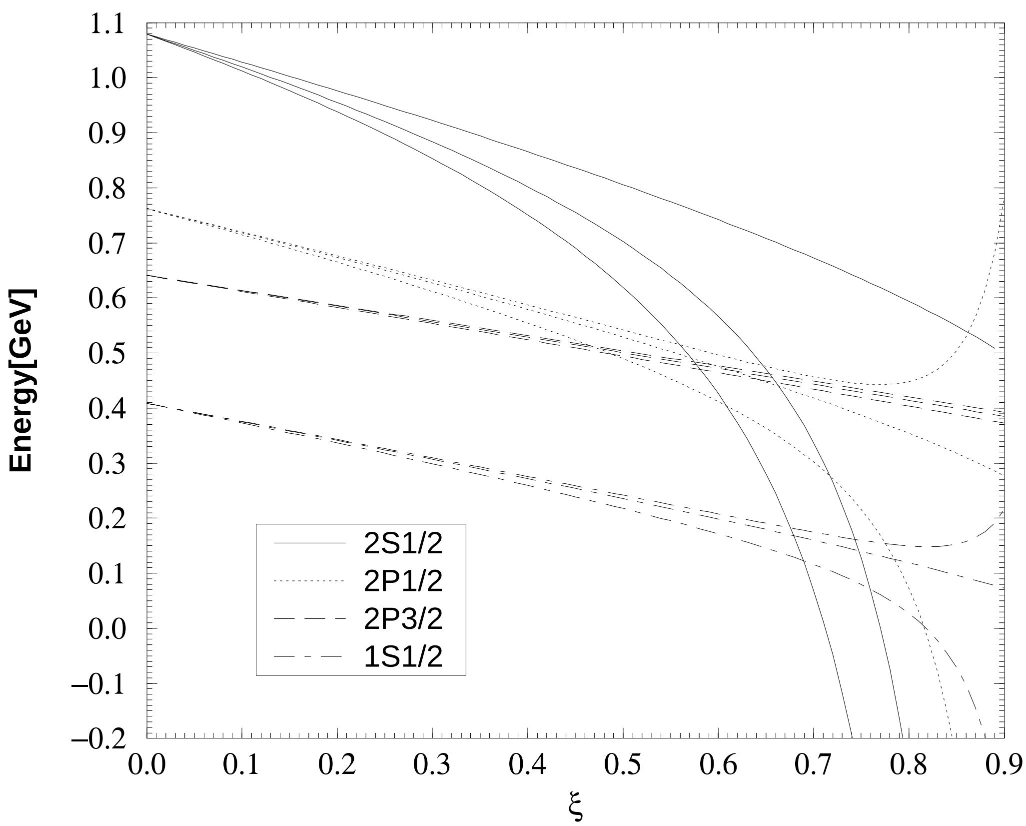

Figure 2: The energy of a (massless) light quark inside a Coulomb bag with radius $R = 5GeV^{-1}$ and a mass of the heavy quark $m_b = 4730MeV$

In figure 2 we have plotted the energy levels for the light quark in the case where $M = m_b$ for constant R. In this graph we have also plotted the heavy quark limit, the energy level above the heavy quark limit is for each pair of states where the spin of the light quark and the heavy quark couples to $S = 1$ and the level below for the case where the spins couples to $S = 0$.

We see that for a given heavy quark multiplet, the induced splitting of the formerly degenerate states with the same angular momentum J of the light quark, but with different angular momentum F for the meson, is a highly nonlinear function of α_s. Only for α_s smaller than 0.2 can the hyperfine splitting with a reasonable approximation be taken as a linear function of α_s as it is in the nonrelativistic quark model treatment.

From figure 2 we also note that the hyperfine splitting (for finite ξ) increases as we go up in light quark excitations. This is quite opposite from the situation in the hydrogen atom and is a reflection of the bag models abrupt confinement.

For the charm sector only for the two lowest states it is reasonable to calculate the hyperfine

splitting perturbatively, this is because of the much smaller mass of the c-quark.

3 The mass functional for heavy mesons

In the previous section we looked at the qualitatively features of the spectra of the heavy mesons. Now we will try to reproduce the quantitatively measured masses. In figure 3 and 4 we have showed the observed spectra of the D and B mesons. The mass formula for heavy mesons in our model is :

$$M = \frac{4}{3}BR^3 + \frac{C}{R} + m_Q + \sqrt{{p_q}^2 + m_q} + \frac{8}{3}\frac{\alpha_s}{M}\frac{\kappa}{4\kappa^2 - 1}(F(F+1) - j(j+1) - \frac{3}{4})\mathcal{I} \tag{17}$$

where $\mathcal{I}$ is an integral over the radial wavefunctions

$$\mathcal{I} = \frac{1}{N}\int_0^R dr(2 + \frac{r^3}{R^3})f(r)^* g(r) \tag{18}$$

When calculating the Coulomb potential in classical electrodynamics it is common to choose

$$V(r) = 0 \qquad \text{for} \qquad r \longrightarrow \infty \tag{19}$$

Inspired by this we adjust the potential inside the bag to be zero at the surface of the bag [12]:

$$V(r) = -\frac{\xi}{r} \longrightarrow -\xi(\frac{1}{r} - \frac{1}{R}) \tag{20}$$

Now the value of the potential at the bag surface will be zero, independent of the radii of the different mesons. Because of this transformation the light quark gets a contribution ξ/R to the energy. This we include in the light quark energy.

The second term, C/R, in equation (17) is supposed to represent the zero point energy in the bag. When one quantizes a radiation field there will always be an infinite zero point energy term, but since physical quantities often are energy differences, the zero point energy falls out. However, when the quantization is carried out in a finite cavity, as in our model, there will be additional pieces of the zero point energy which depends on the size of the cavity. This is represented by the term C/R. The constant C can be calculated if we believe that the zero point energy give rise to the Casimir energy. The Casimir energy inside a perfectly uncharged spherical shell has been calculated by K.A.Milton, L.L.DeRaad, Jr. and J.Schwinger in [13]. They obtained a value $E = 0.09235/(2R)$. To find the value in our model we simply have to multiply the value by eight, because there are eight gluonic radiation fields :

$$C \simeq 0.37 \tag{21}$$

We have determined the masses by minimizing the mass functional by the relation :

$$\frac{\partial M}{\partial R} = 0 \tag{22}$$

It turns out that the integral in equation (18) goes as

$$\frac{1}{m_Q R^2} \tag{23}$$

Since $p_q \sim 1/R$ it is clear that when $R \longrightarrow 0$, $\mathcal{I}$ will dominate. If the factor in front of the hyperfine splitting is negative, equation (17) will for some choice of the parameters have no finite minima. When the radius become small we can not neglect the repulsion of the heavy quark and the model becomes meaningless. This is a well known problem and the usual way of dealing with this is to argue that the divergence will disappear when we calculate higher orders correction. The procedure then is to minimize the energy with respect to R *before* adding the hyperfine term [14]. In our model states with same J and L will then have equal radii.

4 Heavy meson masses

First of all we have to determine the parameters in the model. There are four parameters the strong coupling constant(α_s), the bag constant(B), the heavy meson mass(m_Q) and the light quark mass(m_q). There are of course many ways of determining the parameters, we have chosen too look at the B mesons. We have chosen to determine α_s, B, m_b from the observed masses of the B, B^* and B' mesons, and since the u and d quark have a small mass we have assigned to them a (rather unimportant) mass of $10MeV$. The calculated results are shown in table 1

Experimental Results		Theoretical Results				
	Mass$[MeV]$	Mass$[MeV]$	F^P	State	Radius$[GeV^{-1}]$	μ_m/μ_N
B	5279.4 ± 2.2 [15]	5279	0^-	$1S_{1/2}$	3.94	0
B^*	5324.8 ± 1.8 [15]	5325	1^-	$1S_{1/2}$	3.94	$1.52e_q + 0.203e_Q$
B_0	?	5592	0^+	$2P_{1/2}$	4.50	0
B_1	?	5671	1^+	$2P_{1/2}$	4.50	$0.625e_q + 0.203e_Q$
B_1	5725 [16]	5623	1^+	$2P_{3/2}$	4.47	$1.93e_q + 0.101e_Q$
B_2	5737 [16]	5637	2^+	$2P_{3/2}$	4.47	$2.33e_q + 0.203e_Q$
B'	5859 [17]	5859	0^-	$2S_{1/2}$	4.90	0
B'^*	?	5967	1^-	$2S_{1/2}$	4.90	$0.597e_q + 0.203e_Q$

Table 1: The parameters are $B^{1/4} = 161MeV, \xi = 0.538$ $(\alpha_s = 0.404), m_u = m_d = 10MeV$ and $m_b = 4627MeV.e_q = e_Q = -1/3$ for d and b quarks and $e_q = e_Q = 2/3$ for u and c quarks.

In table 1 there are only given uncertainties for the $1S_{1/2}$ states. This is because the value of the masses for the $2P_{3/2}$ and the lowest $2S_{1/2}$ states was found in [16] and [17] as a fit to the experimental data and it is not quite clear how to put errors to these numbers.

Unfortunately we see that the calculated masses for the B_1 and B_2 mesons are almost exactly $100MeV$ below the experimental values. However, we see that the splitting between the two $P_{3/2}$ mesons is quite well described. One may wonder if there are another set of parameters which will give us a good fit to all the experimental masses listed in table 1. We believe that this is not so. The parameter in the system which has most influence on the spectra is ξ. In figure 1 we plotted the energy levels for ξ between 0 and 0.9. The main problem is that the P states does not move much in direction of the $2S_{1/2}$ state when ξ increase. The boundary condition (8) clearly gives a wrong level splitting. A hope for the model would be that the state we used as B' is not the right one. If we used the B_1 and B_2 states (together with B and B^*) as a normalization the resulting B' would have a mass around $6020MeV$.

Now we have the value of the strong coupling, the bag constant and the b quark mass. In order to calculate the D and B_s meson masses we have to find the c and s quark masses. This has been done by demanding that :

$$m(D_{Exp.}) - m(D_{Teor.}) \approx m(D^*_{Theor}) - m(D^*_{Exp.}) \tag{24}$$

and the s-quark mass by demanding that:

$$m(B_{s\,Exp.}) - m(B_{s\,Theor.}) \approx m(B^*_{s\,Theor.}) - m(B^*_{s\,Exp.}) \tag{25}$$

The results are listed in table 2.

5 Decays

We have looked at electromagnetic and pionic transitions between mesons listed in table 1 and 2. The pionic transitions are calculated using the surface coupling version of the chiral bag model[18].

Experimental Results		Theoretical Results				
	Mass$[MeV]$	Mass$[MeV]$	F^P	State	Radius$[GeV^{-1}]$	μ_m/μ_N
D	1866.9 ± 0.7 [15]	1854	0^-	$1S_{1/2}$	3.94	0
D^*	2008.4 ± 0.7 [15]	2022	1^-	$1S_{1/2}$	3.94	$1.52e_q + 0.725e_Q$
B_s	5369.6 ± 2.4 [15]	5363	0^-	$1S_{1/2}$	3.91	0
B_s^*	5416.3 ± 3.3 [15]	5424	1^-	$1S_{1/2}$	3.91	$1.52e_q + 0.203e_Q$
B_{0_s}	?	5667	0^+	$2P_{1/2}$	4.47	0
B_{1_s}	?	5737	1^+	$2P_{1/2}$	4.47	$0.619e_q + 0.203e_Q$
B_{1_s}	5874 [16]	5718	1^+	$2P_{3/2}$	4.45	$1.93e_q + 0.101e_Q$
B_{2_s}	5886 [16]	5732	2^+	$2P_{3/2}$	4.45	$2.31e_q + 0.203e_Q$
B_s'	?	5887	0^-	$2S_{1/2}$	4.87	0
$B_s'^*$	?	6008	1^-	$2S_{1/2}$	4.87	$0.593e_q + 0.203e_Q$

Table 2: The parameters are $B^{1/4} = 161MeV, \xi = 0.538$ $(\alpha_s = 0.404), m_b = 4627MeV$, $m_c = 1294MeV$ and $m_s = 231MeV$. $e_q = e_Q = -1/3$ for d and b quarks and $e_q = e_Q = 2/3$ for u and c quarks.

In this model a pion field carries the axial current outside the bag produced by the light (u or d) quark inside the bag. This makes the model chirally symmetric for massless u and d quarks and the interaction between the bag and the pion field is given by :

$$\mathcal{L}_{\text{int}} = -\frac{i}{2f_\pi}\overline{\psi}\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\phi}\psi\Delta_s, \tag{26}$$

τ_i are the Pauli isospin matrices and $\boldsymbol{\phi}$ an isovector representing the pion field. Δ_s is a covariant surface delta-function.

The calculation of transitions involving pions is then straight forward, some expressions for pionic transitions are listed below :

$$0^- \longrightarrow 1^-0^- \begin{cases} V_{fi} = \frac{-i}{f_\pi}\sqrt{\frac{\pi}{3V\omega_{\boldsymbol{k}}}}P(R)\left(Y_1^{-1}(\hat{\boldsymbol{k}}) + Y_1^1(\hat{\boldsymbol{k}}) + Y_1^0(\hat{\boldsymbol{k}})\right)C_\pi \\ \Gamma(B' \longrightarrow B^*\pi) = \frac{1}{4\pi}\frac{k}{f_\pi^2}|P(R)C_\pi|^2 \end{cases} \tag{27}$$

$$0^- \longrightarrow 0^+0^- \begin{cases} V_{fi} = \frac{-1}{f_\pi}\sqrt{\frac{\pi}{V\omega_{\boldsymbol{k}}}}S(R)Y_0^0(\hat{\boldsymbol{k}})C_\pi \\ \Gamma(B' \longrightarrow B_0\pi) = \frac{1}{4\pi}\frac{k}{f_\pi^2}|S(R)C_\pi|^2 \end{cases} \tag{28}$$

$$2^+ \longrightarrow 0^-0^- \begin{cases} V_{fi} = -\frac{1}{f_\pi}\sqrt{\frac{2\pi}{5V\omega_{\boldsymbol{k}}}}D(R)Y_2^2(\hat{\boldsymbol{k}})C_\pi \\ \Gamma(B_2 \longrightarrow B\pi) = \frac{1}{10\pi}\frac{k}{f_\pi^2}|D(R)C_\pi|^2 \end{cases} \tag{29}$$

$$2^+ \longrightarrow 1^-0^- \begin{cases} V_{fi} = \frac{1}{f_\pi}\sqrt{\frac{2\pi}{5V\omega_{\boldsymbol{k}}}}D(R)\left(Y_2^2(\hat{\boldsymbol{k}}) - \frac{1}{\sqrt{2}}Y_2^1(\hat{\boldsymbol{k}})\right)C_\pi \\ \Gamma(B_2 \longrightarrow B^*\pi) = \frac{3}{20\pi}\frac{k}{f_\pi^2}|D(R)C_\pi|^2 \end{cases} \tag{30}$$

$$1^+ \longrightarrow 1^-0^- \begin{cases} V_{fi} = \frac{1}{f_\pi}\sqrt{\frac{\pi}{10V\omega_{\boldsymbol{k}}}}D(R)\left(\sqrt{6}Y_2^2(\hat{\boldsymbol{k}}) - \sqrt{3}Y_2^1(\hat{\boldsymbol{k}}) + Y_2^0(\hat{\boldsymbol{k}})\right)C_\pi \\ \Gamma(B_1(P_{3/2}) \longrightarrow B^*\pi) = \frac{1}{4\pi}\frac{k}{f_\pi^2}|D(R)C_\pi|^2 \end{cases} \tag{31}$$

$$1^+ \longrightarrow 1^-0^- \begin{cases} V_{fi} = \frac{1}{f_\pi}\sqrt{\frac{\pi}{V\omega_{\boldsymbol{k}}}}S(R)Y_0^0(\hat{\boldsymbol{k}})C_\pi \\ \Gamma(B_1(P_{1/2}) \longrightarrow B^*\pi) = \frac{1}{4\pi}\frac{k}{f_\pi^2}|S(R)C_\pi|^2 \end{cases} \tag{32}$$

$$0^+ \longrightarrow 0^-0^- \begin{cases} V_{fi} = -\frac{1}{f_\pi}\sqrt{\frac{\pi}{V\omega_{\boldsymbol{k}}}}S(R)Y_0^0(\hat{\boldsymbol{k}})C_\pi \\ \Gamma(B_0 \longrightarrow B\pi) = \frac{1}{4\pi}\frac{k}{f_\pi^2}|S(R)C_\pi|^2 \end{cases} \tag{33}$$

$J_\alpha^{P\alpha} \longrightarrow J_\beta^{P\beta}\pi$	$\alpha \longrightarrow \beta\pi$	Teor.$[MeV]$	$g_{\alpha\beta\pi}$
$0^- \longrightarrow 1^-\pi$	$B' \longrightarrow B^*\pi^+$	84.3	42.8
	$B' \longrightarrow B^*\pi^0$	42.2	30.1
$0^- \longrightarrow 0^+\pi$	$B' \longrightarrow B_0\pi^+$	45.3	23.0
	$B' \longrightarrow B_0\pi^0$	22.7	16.2
$2^+ \longrightarrow 1^-\pi$	$B_2 \longrightarrow B^*\pi^+$	9.17	55.8
	$B_2 \longrightarrow B^*\pi^0$	4.67	39.4
$2^+ \longrightarrow 0^-\pi$	$B_2^+ \longrightarrow B^0\pi^+$	9.74	43.5
	$B_2^0 \longrightarrow B^0\pi^0$	4.94	30.7
	$B_2^+ \longrightarrow B^+\pi^0$	4.98	30.7
	$B_2^0 \longrightarrow B^+\pi^-$	9.82	43.5
$1^+(P_{3/2}) \longrightarrow 1^-\pi$	$B_1 \longrightarrow B^*\pi^+$	13.3	72.8
	$B_1 \longrightarrow B^*\pi^0$	6.71	51.1
$1^+(P_{1/2}) \longrightarrow 1^-\pi$	$B_1 \longrightarrow B^*\pi^+$	92.7	27.1
	$B_1 \longrightarrow B^*\pi^0$	46.1	19.0
$0^+ \longrightarrow 0^-\pi$	$B_0^+ \longrightarrow B^0\pi^+$	93.7	28.5
	$B_0^0 \longrightarrow B^0\pi^0$	46.8	20.1
	$B_0^+ \longrightarrow B^+\pi^0$	46.9	20.1
	$B_0^0 \longrightarrow B^+\pi^-$	93.7	28.5
$1^- \longrightarrow 0^-\pi$	$D^{*+} \longrightarrow D^0\pi^+$	6.01 10^{-2}	17.1
	$D^{*+} \longrightarrow D^+\pi^0$	2.72 10^{-2}	12.1
	$D^{*0} \longrightarrow D^0\pi^0$	3.88 10^{-2}	12.1

Table 3: Pion decays

$$1^- \longrightarrow 0^-0^- \begin{cases} V_{fi} = \frac{-i}{f_\pi}\sqrt{\frac{\pi}{3V\omega_{\boldsymbol{k}}}}P(R)Y_1^1(\hat{\boldsymbol{k}})C_\pi \\ \Gamma(D^* \longrightarrow D\pi) = \frac{1}{12\pi}\frac{k}{f_\pi^2}|P(R)C_\pi|^2. \end{cases} \tag{34}$$

In these formulae :

$$S(R) \equiv R^2\left(f_\beta^*(R)g_\alpha(R) + g_\beta^*(R)f_\alpha(R)\right)j_0(kR) \qquad \text{(S-wave.)} \tag{35}$$
$$P(R) \equiv R^2\left(f_\beta^*(R)g_\alpha(R) + g_\beta^*(R)f_\alpha(R)\right)j_1(kR) \qquad \text{(P-wave.)} \tag{36}$$
$$D(R) \equiv R^2\left(f_\beta^*(R)g_\alpha(R) + g_\beta^*(R)f_\alpha(R)\right)j_2(kR) \qquad \text{(D-wave.)} \tag{37}$$

The index $\alpha(\beta)$ refers to the initial(final) meson.

$$C_\pi \equiv \begin{cases} 1 & \text{for} \quad \pi^\pm \\ \frac{1}{\sqrt{2}} & \text{for} \quad \pi^0. \end{cases} \tag{38}$$

The above transition rates (27)-(34) have been numerically calculated and are listed in table 3.

In addition to the partial widths listed in table 3 we have also calculated the coupling constants (to the right in table 3), by using the following definition :

$$g_{\alpha\beta\pi} \equiv \sqrt{\frac{\Gamma(\alpha \longrightarrow \beta\pi)24\pi M_\alpha^2}{k^{2L+1}}} \tag{39}$$

k is the pion momenta and M_α the mass of the decaying particle. The dimension of the couplings goes as $[GeV]^{-L+1}$, where L is the relative angular momentum between the decay products. Only $L = 1$ transitions such as $1^- \longrightarrow 0^-\pi$ are then dimension less by the definition (39).

$J^P_\alpha \longrightarrow J^P_\beta \pi$	$\alpha \longrightarrow \beta\pi$	Teor.$[keV]$
$0^- \longrightarrow 2^+ + 1^-$	$B^{**0} \longrightarrow B_2^0\gamma$	2.62 10^{-3}
	$B^{**+} \longrightarrow B_2^+\gamma$	1.05 10^{-2}
$0^- \longrightarrow 1^+ + 1^-$	$B^{**0} \longrightarrow B_1^0\gamma$	1.77 10^{1}
	$B^{**+} \longrightarrow B_1^+\gamma$	7.07 10^{1}
$0^- \longrightarrow 1^- + 1^-$	$B^{**0} \longrightarrow B^{*0}\gamma$	1.20 10^{-1}
	$B^{**+} \longrightarrow B^{*+}\gamma$	7.89 10^{-1}
$0^- \longrightarrow 0^- + 1^-$	$B^{**0} \longrightarrow B^0\gamma$	0
	$B^{**+} \longrightarrow B^+\gamma$	0
$2^+ \longrightarrow 1^+ + 1^-$	$B_2^0 \longrightarrow B_1^0\gamma$	6.26 10^{-2}
	$B_2^+ \longrightarrow B_1^+\gamma$	2.50 10^{-3}
$2^+ \longrightarrow 1^- + 1^-$	$B_2^0 \longrightarrow B^{*0}\gamma$	6.76 10^{1}
	$B_2^+ \longrightarrow B^{*+}\gamma$	2.70 10^{2}
$2^+ \longrightarrow 0^- + 1^-$	$B_2^0 \longrightarrow B^0\gamma$	4.08
	$B_2^+ \longrightarrow B^+\gamma$	1.63 10^{1}
$2^+ \longrightarrow 1^+ + 1^-$	$B_{s2}^0 \longrightarrow B_{s1}^0\gamma$	3.38 10^{-4}
$2^+ \longrightarrow 1^- + 1^-$	$B_{s2}^0 \longrightarrow B_s^{*0}\gamma$	5.86 10^{1}
$2^+ \longrightarrow 0^- + 1^-$	$B_{s2}^0 \longrightarrow B_s^0\gamma$	4.62
$1^+ \longrightarrow 1^- + 1^-$	$B_1^0 \longrightarrow B^{*0}\gamma$	2.73 10^{1}
	$B_1^+ \longrightarrow B^{*+}\gamma$	1.09 10^{2}
$1^+ \longrightarrow 0^- + 1^-$	$B_1^0 \longrightarrow B^0\gamma$	4.10 10^{1}
	$B_1^+ \longrightarrow B^+\gamma$	1.64 10^{2}
$1^+ \longrightarrow 1^- + 1^-$	$B_{s1}^0 \longrightarrow B_s^{*0}\gamma$	2.56 10^{1}
$1^+ \longrightarrow 0^- + 1^-$	$B_{s1}^0 \longrightarrow B_s^0\gamma$	3.46 10^{1}
$1^- \longrightarrow 0^- + 1^-$	$B^{*0} \longrightarrow B^0\gamma$	6.41 10^{-2}
	$B^{*+} \longrightarrow B^+\gamma$	2.72 10^{-1}
	$B_s^{*0} \longrightarrow B_s^0\gamma$	5.10 10^{-2}
	$D^{*0} \longrightarrow D^0\gamma$	7.18
	$D^{*+} \longrightarrow D^+\gamma$	1.73

Table 4: Photon decays

It is also possible to calculate the coupling $g_{B^*B\pi}$, the B^* emits a virtual pion. This coupling has been calculated in the rest system of the heavy meson at zero recoil, the result is :

$$g_{B^*B\pi^\pm} = \sqrt{\frac{2}{9}}\frac{R}{f_\pi}|R^2\left(f^*_\beta(R)g_\alpha(R) + g^*_\beta(R)f_\alpha(R)\right)|M_{B^*} \tag{40}$$

$$g_{B^*B\pi^0} = \sqrt{\frac{1}{9}}\frac{R}{f_\pi}|R^2\left(f^*_\beta(R)g_\alpha(R) + g^*_\beta(R)f_\alpha(R)\right)|M_{B^*}. \tag{41}$$

Using the wavefunctions for the B^* and B meson give us

$$g_{B^*B\pi^+} = 45.6 \tag{42}$$

$$g_{B^*B\pi^0} = 32.2. \tag{43}$$

The coupling of photons to the mesonic states are done straight forward by using the interaction Lagrangian :

$$\mathcal{L}_{\text{int}} = e_q e\overline{\psi}\boldsymbol{\gamma}\cdot\boldsymbol{A}\psi \tag{44}$$

Some expressions for electromagnetic transitions are listed below :

$$0^- \longrightarrow 2^+1^- \left\{ \Gamma = \frac{24}{5}\alpha e_q^2 \omega_{\boldsymbol{k}} \left| \int dx j_2(F+G) \right|^2 \right. \tag{45}$$

$$0^- \longrightarrow 1^+1^- \left\{ \begin{aligned} \Gamma = \; & \frac{8}{3}\alpha e_q^2 \omega_{\boldsymbol{k}} \left(\frac{1}{5} \left| \int dx j_2(F+G) \right|^2 \right. \\ & \left. + \left| \int dx (Fj_2 - Gj_0) \right|^2 + \frac{1}{3} \left| \int dx (Gj_2 + Gj_0) \right|^2 \right) \end{aligned} \right. \tag{46}$$

$$0^- \longrightarrow 1^-1^- \left\{ \Gamma = \frac{24}{5}\alpha e_q^2 \omega_{\boldsymbol{k}} \left| \int dx j_1(F+G) \right|^2 \right. \tag{47}$$

$$0^- \longrightarrow 0^-1^- \left\{ \Gamma = 0 \right. \tag{48}$$

$$2^+ \longrightarrow 1^+1^- \left\{ \begin{aligned} \Gamma = \; & \frac{2}{375}\alpha e_q^2 \omega_{\boldsymbol{k}} \left(\frac{87}{4} \left| \int dx j_1(F+G) \right|^2 \right. \\ & +52 \left| \int dx (Gj_3 + Gj_1) \right|^2 + 2 \left| \int dx (Gj_3 - Fj_1) \right|^2 \\ & +27 \left| \int dx (Fj_3 - Gj_1) \right|^2 + 27 \left| \int dx (Fj_3 + Fj_1) \right|^2 \\ & \left. + \frac{3501}{7} \left| \int dx j_3(F+G) \right|^2 \right) \end{aligned} \right. \tag{49}$$

$$2^+ \longrightarrow 1^-1^- \left\{ \begin{aligned} \Gamma = \; & \frac{4}{3}\alpha e_q^2 \omega_{\boldsymbol{k}} \left(\frac{11}{10} \left| \int dx j_2(F+G) \right|^2 + \left| \int dx (Gj_2 - Fj_0) \right|^2 \right. \\ & \left. + \frac{1}{3} \left| \int dx (Fj_2 + Fj_0) \right|^2 \right) \end{aligned} \right. \tag{50}$$

$$2^+ \longrightarrow 0^-1^- \left\{ \Gamma = \frac{6}{5}\alpha e_q^2 \omega_{\boldsymbol{k}} \left| \int dx j_2(F+G) \right|^2 \right. \tag{51}$$

$$1^+ \longrightarrow 1^-1^- \left\{ \begin{aligned} \Gamma = \; & \frac{2}{9}\alpha e_q^2 \omega_{\boldsymbol{k}} \left(13 \left| \int dx j_2(F+G) \right|^2 + \frac{2}{3} \left| \int dx (Fj_2 + Fj_0) \right|^2 \right. \\ & \left. +2 \left| \int dx (Gj_2 - Fj_0) \right|^2 \right) \end{aligned} \right. \tag{52}$$

$$1^+ \longrightarrow 0^-1^- \left\{ \begin{aligned} \Gamma = \; & \frac{8}{9}\alpha e_q^2 \omega_{\boldsymbol{k}} \left(-\frac{1}{4} \left| \int dx j_2(F+G) \right|^2 + \frac{1}{3} \left| \int dx (Fj_2 + Fj_0) \right|^2 \right. \\ & \left. + \left| \int dx (Gj_2 - Fj_0) \right|^2 \right) \end{aligned} \right. \tag{53}$$

$$1^- \longrightarrow 0^-1^- \left\{ \Gamma = \frac{4}{3}\alpha e_q^2 \omega_{\boldsymbol{k}} \left| \int dx j_1(F+G) \right|^2 \right. \tag{54}$$

where we have defined :

$$F \equiv x^2 f_\beta(x)^* g_\alpha(x) \tag{55}$$

$$G \equiv x^2 g_\beta(x)^* f_\alpha(x) \tag{56}$$

$$j_l \equiv j_l(\omega_{\boldsymbol{k}} x) \tag{57}$$

The numerical values for the above expressions are shown in table 4. The transitions in table 4 are very suppressed in comparison with those listed in table 3. This of course is expected, the smaller phase space for pion decays is compensated by the much stronger pion coupling relative to the electromagnetic coupling.

5.1 Comparison with theoretical and experimental results

We have calculated a lot of partial widths for different particles, listed in table 3 and 4. To day very little is known on the experimental front, but there are a lot of theoretical predictions. So we will compare our results with the known experimental and some of the theoretical results. As we shall see there are no conflict between our predictions and the experimental information.

In table 5 we have listed some theoretical and experimental results, on the experimental limits we have assumed that the the width of D^{*0} have the same upper limit as the width of $D^{*\pm}$. It may not be clear from table 5, but the theoretical predictions vary a lot. In [22] there is a summary of theoretical estimates. For the particular decay $D^{*+} \longrightarrow D^0\pi^+$ that determines the coupling constant $g_{D^*D\pi}$, the predicted rates vary from $10keV$(QCD sum rules) to more than $100keV$(quark model + chiral HQET). We obtained $\Gamma(D^{*+} \longrightarrow D^0\pi^+) = 60.1keV$, close to the value $(61-78keV)$ coming from P.Cho and H.Georgi [23] who makes calculations with chiral HQET. The value of the coupling

	Theoretical				Experimental
	[19](MeV)	[20](keV)	[21](keV)	our work	[15](keV)
$\Gamma(B_2 \longrightarrow B^*\pi)$	11	$-$	$-$	$13.8MeV$	$-$
$\Gamma(B_2 \longrightarrow B\pi)$	10	$-$	$-$	$14.7MeV$	$-$
$\Gamma(B_1 \longrightarrow B^*\pi)$	14	$-$	$-$	$20.0MeV$	$-$
$\Gamma(B^{*+} \longrightarrow B\gamma)$	$-$	$-$	0.38 ± 0.06	$0.272keV$	$-$
$\Gamma(B^{*0} \longrightarrow B^0\gamma)$	$-$	$-$	0.13 ± 0.03	$6.41\ 10^{-2}keV$	$-$
$\Gamma(B_s^* \longrightarrow B_s\gamma)$	$-$	$-$	0.22 ± 0.04	$5.10\ 10^{-2}keV$	$-$
$\Gamma(D^{*+} \longrightarrow D^0\pi^+)$	$-$	69.1	$-$	$60.1keV$	< 91
$\Gamma(D^{*+} \longrightarrow D^+\pi^0)$	$-$	32.1	$-$	$27.2keV$	< 44
$\Gamma(D^{*0} \longrightarrow D^0\pi^0)$	$-$	46.0	$-$	$38.8keV$	< 85
$\Gamma(D^{*+} \longrightarrow D^+\gamma)$	$-$	0.919	0.23 ± 0.1	$1.72keV$	< 4.1
$\Gamma(D^{*0} \longrightarrow D^0\gamma)$	$-$	23.5	12.9 ± 2	$7.18keV$	< 54

Table 5: Comparison with theoretical and experimental results

$g_{B^*B\pi^+}$ vary from $g_{B^*B\pi^+} = 15 \pm 4$(QCD sum rules) to $g_{B^*B\pi^+} = 64$(quark model+chiral HQET), we found $g_{B^*B\pi^+} = 45.6$.

We have calculated most but not all decay modes for the excited states. The $\pi\pi$ modes are missing. As we believe that these modes are less important than the emission of single pions in the decays we still can give approximate values of the decay widths, these are :

$$2S_{1/2} \ :\ \Gamma(B') \simeq 195MeV \tag{58}$$
$$2P_{3/2} \ :\ \Gamma(B_2) \simeq 29MeV \qquad \Gamma(B_1) \simeq 20MeV \tag{59}$$
$$2P_{1/2} \ :\ \Gamma(B_1) \simeq 139MeV \qquad \Gamma(B_0) \simeq 141MeV \tag{60}$$
$$1S_{1/2} \ :\ \Gamma(D^{*+}) \simeq 89keV \qquad \Gamma(D^{*0}) \simeq 46keV \tag{61}$$

The $P_{1/2}$ states are naturally much wider than the $P_{3/2}$ states because they decay only trough an S-wave, whereas the $P_{3/2}$ states decay through a D-wave. The full width of the P states indicated by a preliminary experiment [24] are $\Gamma(B(P_{3/2})) \simeq 20MeV$ and $\Gamma(B(P_{1/2})) \simeq 150MeV$. This is, as we see, in good agreement with our results. Since the $P_{1/2}$ states are so broad, they are very hard to reconstruct from the experimental results, and so far there have not been any really precise measurement of their masses.

The CLEO report [25] contains best measurement of the D^{*+} branching fractions, a large improvement of what can be found in the Particle Data Book [15]. Since we have calculated the width of the D^{*+} meson, equation (61), it is easy to calculate the branching ratios. The results are shown in table 6 together with the CLEO results. Our results are clearly in good agreement with the experimental data We recognize however that branching ratios are one thing, particular decay widths another. We get the correct ratio between pionic and electromagnetic decays. As the coupling of the electromagnetic field to the quarks is simple, we naturally have some confidence in the calculated electromagnetic transitions rates. Therefore we believe that our calculated pionic rates cannot be too far off from what will be measured in the future.

	CLEO	our calculations
$B_r(D^{*+} \longrightarrow D^+\gamma)$	$(1.68 \pm 0.51)\%$	1.94%
$B_r(D^{*+} \longrightarrow D^+\pi^0)$	$(30.73 \pm 0.63)\%$	30.55%
$B_r(D^{*+} \longrightarrow D^+\pi^+)$	$(67.59 \pm 0.70)\%$	67.51%

Table 6: Comparison with CLEO data and our predictions

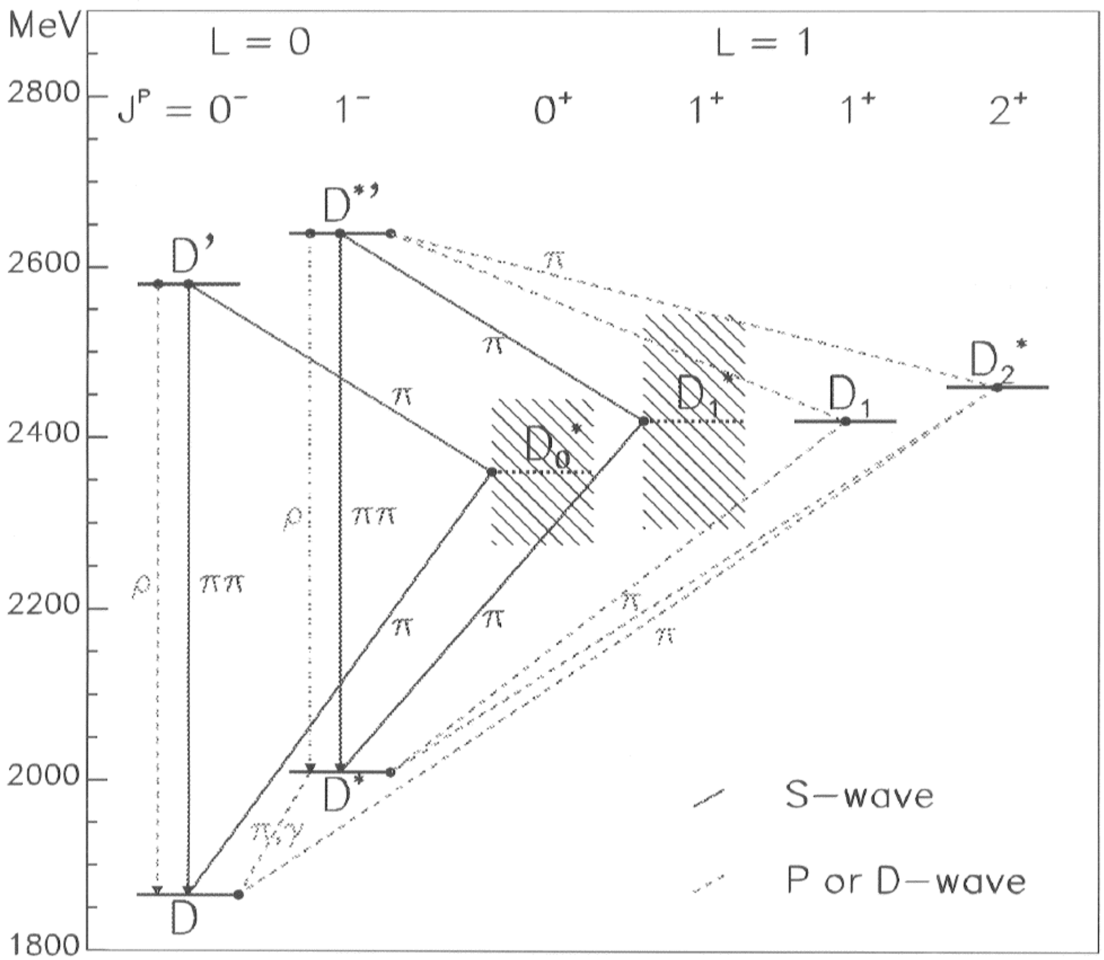

Figure 3: D meson spectra and transition lines.

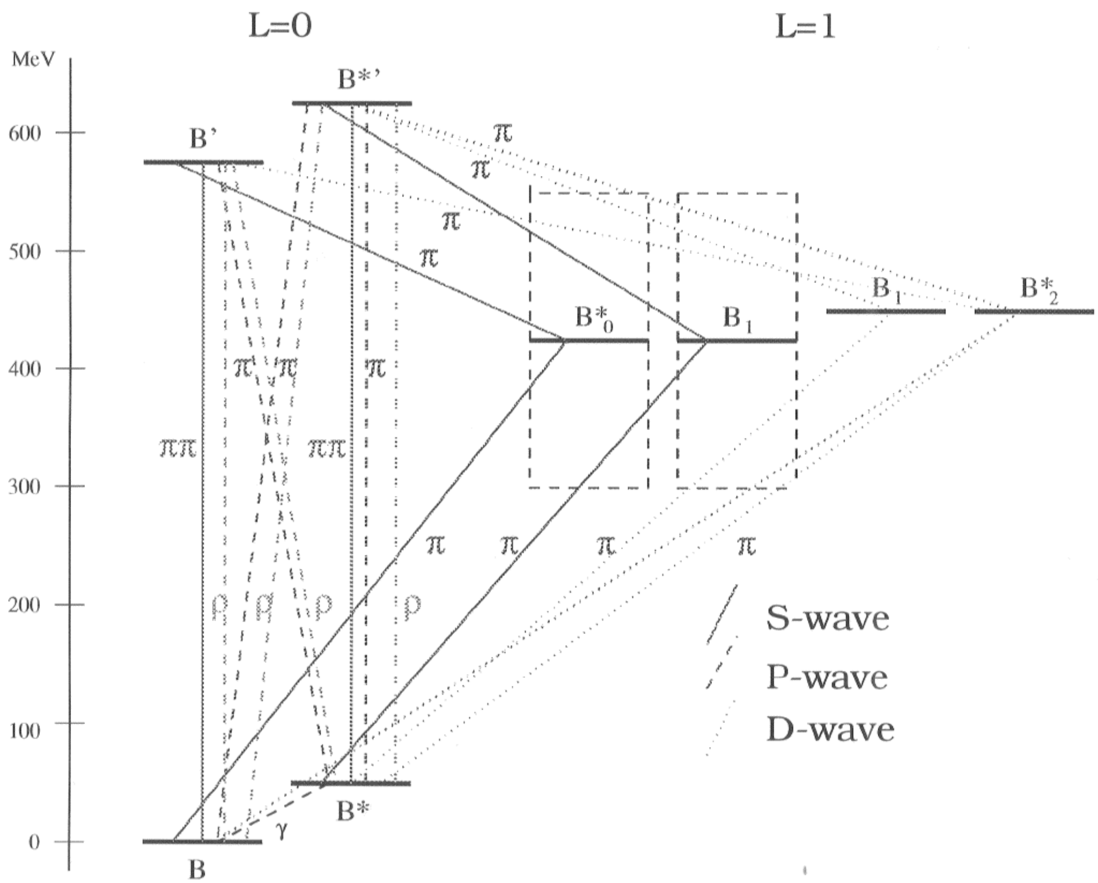

Figure 4: B meson spectra and transition lines.

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